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Effective U-values (R-values) for Tapered Insulation

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Introduction

As energy codes become more stringent and better enforced it will become more important to properly calculate effective roof assembly U-values (R-values) for compliance. One area where effective U-values are commonly misunderstood is tapered insulation for low slope roofs (Figure 1) or at the eaves of sloped roofs. This bulletin presents the theory and calculation methods used to develop tables of effective R-values for tapered insulation.

Figure 1: Tapered EPS insulation below polyisocyanurate insulation in a low slope roof application.

R-Values, U-Values, and Heat Flow (Q)

R-value and U-value are both a function of insulation thickness, and this thickness varies in tapered insulation systems. R-value varies linearly with insulation thickness and thus horizontal distance (Figure 2) in a tapered insulation plan. However, U-value varies as an inverse function of insulation thickness, and therefore it is non-linear (Figure 3). The heat flow (Q) through an assembly varies proportionally with the U-value. The function for heat flow (Q) is:

$$Q = UA\Delta T = \frac{1}{R} A\Delta T$$

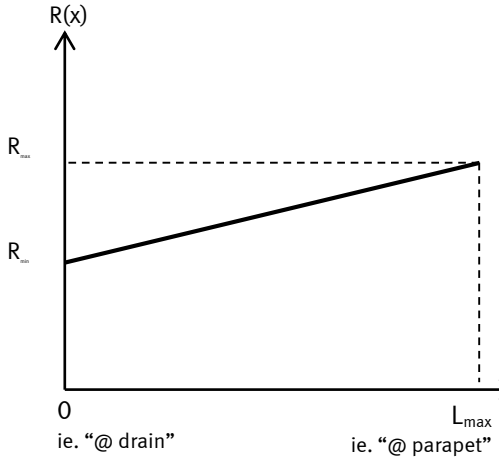


Figure 2 - Plot of R-value vs. horizontal distance in tapered insulation.

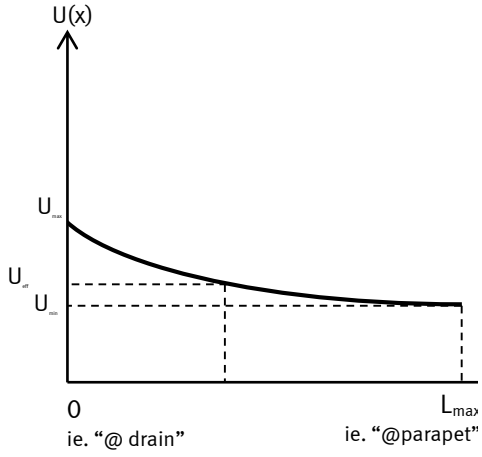


Figure 3 - Plot of U-value vs. horizontal distance in tapered insulation.

This non-linear relationship has the effect of making the estimation of the overall effective U-value (or R-Value) not possible by linear interpolation. In other words, the effective R-value cannot be calculated by taking the value half way between R_{max} and R_{min} and subsequently the effective U-value is not the inverse of this value.

The correct determination of the effective U-value (or R-Value) should yield the value above and below which half of the total heat flow occurs. This bulletin describes the correct method for determining these effective values.

Calculating Effective U-Value of Sloped Insulation

There are two different typical slope configurations of insulation and they require different methods to calculate the effective U-value. The first typical configuration is a single sloped rectangle (Figure 4) and the second is a sloped triangle towards a central drain (Figure 5), similar for slope to exterior edges (pyramid).

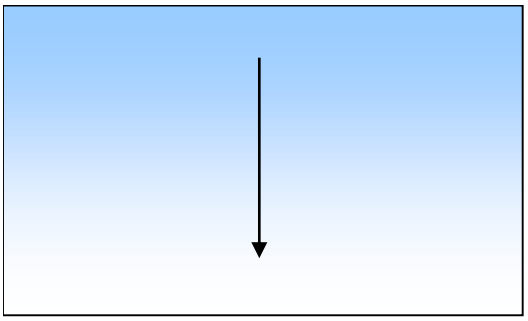


Figure 4: Typical single sloped rectangle.

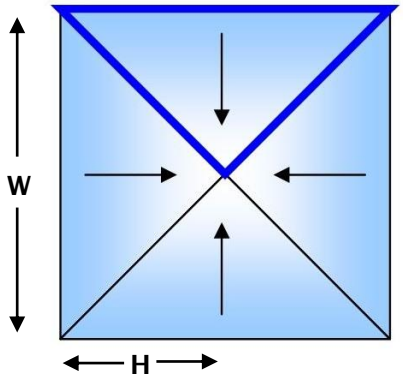


Figure 5: Typical single sloped triangle.

Rectangle Effective U-Value

The effective U-value is determined by finding the total heat flow through a horizontal section through the tapered insulation and then averaging this over the length of the section.

The equation for U-value as a function of horizontal distance is simply the inverse of the R-value function:

$$U(x) = \frac{1}{R(x)}$$

The linear equation for R-value as a function of horizontal distance is:

$$R(x) = \frac{R_{max} - R_{min}}{L_{max}} x + R_{min}$$

Thus:

$$U(x) = \frac{1}{R(x)} = \left[\frac{R_{max} - R_{min}}{L_{max}} x + R_{min} \right]^{-1}$$

The effective U-value (U-effective) is found by calculating the total heat flow averaged over the total tapered cross section:

$$U_{eff} = \frac{\int_0^{L_{max}} U(x)dx}{L_{max}}$$

$$U_{eff} = \frac{1}{L_{max}} \int_0^{L_{max}} \left[\frac{R_{max} - R_{min}}{L_{max}} x + R_{min} \right]^{-1} dx$$

$$U_{eff} = \frac{1}{R_{max} - R_{min}} \ln \left[\frac{R_{max} - R_{min}}{L_{max}} x + R_{min} \right]_{x=0}^{x=L_{max}}$$

$$U_{eff} = \frac{1}{R_{max} - R_{min}} \ln \left[\frac{R_{max}}{R_{min}} \right]$$

R-effective is the inverse of the function for U-effective:

$$R_{eff} = \frac{R_{max} - R_{min}}{\ln \left[\frac{R_{max}}{R_{min}} \right]}$$

Therefore, the effective R-value is based on an inverse logarithmic function and is dependent only on the maximum and minimum R-values. The effective R-value is independent of the length of the tapered cross-section. A table is provided at the end of this bulletin.

Triangle Effective R-Value

Similar to the method for calculating the effective R-value of a sloped rectangle, R-value and U-value as a function of horizontal distance (low point to midpoint of high edge) are:

$$R(x) = \frac{R_{max} - R_{min}}{L_{max}} x + R_{min}$$

$$U(x) = \frac{1}{R(x)} = \left[\frac{R_{max} - R_{min}}{L_{max}} x + R_{min} \right]^{-1}$$

The width of the triangle perpendicular to the horizontal distance is:

$$w(x) = x \times \frac{W}{L}$$

W is the width of the triangle perpendicular to the horizontal distance and H is the height of the triangle parallel to the horizontal distance (Fig 5).

As before, U-effective is found by calculating the total heat flow averaged over the total tapered cross section:

$$U_{eff} = \frac{1}{L}\int_0^L \frac{w(x)}{w_{avg}} U(x)dx$$

The above equation is essentially the same as that for the rectangular slope except that the $w(x)/w_{avg}$ length weights the U-values which accounts for the changing width of the triangle.

By substituting and then completing the algebraic manipulation, the equation becomes:

$$U_{eff} = \frac{W}{L^2 w_{avg}} \int_0^H \frac{x}{\left(\frac{R_{max} - R_{min}}{L}\right)x + R_{min}} dx$$

$$U_{eff} = \frac{2}{R_{max} - R_{min}} \left[1 + \frac{R_{min}}{R_{max} - R_{min}} \ln\left(\frac{R_{min}}{R_{max}}\right)\right]$$

R-effective is the inverse of the function for U-effective:

$$R_{eff} = \left[\frac{2}{R_{max} - R_{min}} \left[1 + \frac{R_{min}}{R_{max} - R_{min}} \ln\left(\frac{R_{min}}{R_{max}}\right)\right]\right]^{-1}$$

Notice again that the dimensions of the wedge have no impact on the effective R-value, only the high and low thickness R-values.

It is also important to note that R_{max} refers to the insulation value along the long edge of the triangle and R_{min} to the insulation at the point of the triangle which assumes that the insulation slopes to the centre. If instead the slope was downward to the edge, R_{max} and R_{min} could be interchanged to correctly calculate R-effective.

Tables are provided at the end of this bulletin for triangle insulation sloped to the center and exterior edges.

Procedure for Using the Table of Effective U-Values (R-Values)

Tables negate the need to undertake the above complex calculations for each application. Use of these tables requires:

- → Determination of maximum and minimum thickness of insulation along linearly tapered cross section (on site measurement, drawings)
- → Determination of R-value/inch of tapered insulating material.
- → Determination of R_{max} and R_{min}.
- → Use the chart to find corresponding effective R-value of insulated area.

To illustrate the necessity of using this method for determining R-effective, a sample of the differing results yielded by the incorrect linear interpolation and the correct heat flow method is provided below.

Table 1 – Sample Comparison of Results in Rectangular Tapered Insulation R-Value Calculation

R_{min}	R_{max}	R-average (linear)	R-effective (correct)
5	20	12.5	**10.8**
20	25	22.5	**22.4**
10	60	35.0	**27.9**

Table 2 – Sample Comparison of Results in Triangular Tapered Insulation R-Value Calculation for Centre Drained

R_{min}	R_{max}	R-average (linear)	R-effective (correct)
5	20	12.5	**13.9**
20	25	22.5	**23.3**
10	60	35.0	**39.0**

For rectangular tapered insulation the linear method consistently overstates the effective R-value; however, for the triangular tapered insulation that drains to the centre the linear method understates the effective R-value. The overstating and understating is most pronounced in instances where there is a large difference between minimum and maximum insulation R-values.

Limitations of Formula/Table Values

- These methods assume linearly tapered insulation that is tapered along only one axis.
- In areas of differing insulation slopes/configurations, individual U-values should be found and an area weighted U-value calculation should be used to determine the effective values.
- The formulas yield only the theoretical effective R-value and do not take into account losses in R-value from physical effects such as rain or thermal bridging.

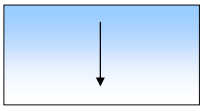
For additional information on this and other topics, please visit our website, rdh.com, or contact us at contact@rdh.com.

Additional Resources

- Try out our R-Value Calculator on our website: http://rdh.com/new-construction/r-value-calculator-app/

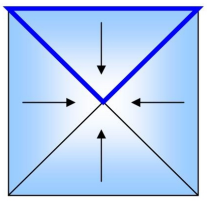
Effective R-Value for Tapered Insulation - Single Slope Rectangular to One-side

Rmin ↓ / Rmax →	1	2	3	4	5	6	7	8	9	10	12	14	16	18	20	25	30	35	40	45	50	55	60
1	1.0	1.4	1.8	2.2	2.5	2.8	3.1	3.4	3.6	3.9	4.4	4.9	5.4	5.9	6.3	7.5	8.5	9.6	10.6	11.6	12.5	13.5	14.4
2	-	2.0	2.5	2.9	3.3	3.6	4.0	4.3	4.7	5.0	5.6	6.2	6.7	7.3	7.8	9.1	10.3	11.5	12.7	13.8	14.9	16.0	17.1
3	-	-	3.0	3.5	3.9	4.3	4.7	5.1	5.5	5.8	6.5	7.1	7.8	8.4	9.0	10.4	11.7	13.0	14.3	15.5	16.7	17.9	19.0
4	-	-	-	4.0	4.5	4.9	5.4	5.8	6.2	6.5	7.3	8.0	8.7	9.3	9.9	11.5	12.9	14.3	15.6	16.9	18.2	19.5	20.7
5	-	-	-	-	5.0	5.5	5.9	6.4	6.8	7.2	8.0	8.7	9.5	10.1	10.8	12.4	14.0	15.4	16.8	18.2	19.5	20.9	22.1
6	-	-	-	-	-	6.0	6.5	7.0	7.4	7.8	8.7	9.4	10.2	10.9	11.6	13.3	14.9	16.4	17.9	19.4	20.8	22.1	23.5
7	-	-	-	-	-	-	7.0	7.5	8.0	8.4	9.3	10.1	10.9	11.6	12.4	14.1	15.8	17.4	18.9	20.4	21.9	23.3	24.7
8	-	-	-	-	-	-	-	8.0	8.5	9.0	9.9	10.7	11.5	12.3	13.1	14.9	16.6	18.3	19.9	21.4	22.9	24.4	25.8
9	-	-	-	-	-	-	-	-	9.0	9.5	10.4	11.3	12.2	13.0	13.8	15.7	17.4	19.1	20.8	22.4	23.9	25.4	26.9
10	-	-	-	-	-	-	-	-	-	10.0	11.0	11.9	12.8	13.6	14.4	16.4	18.2	20.0	21.6	23.3	24.9	26.4	27.9
12	-	-	-	-	-	-	-	-	-	-	12.0	13.0	13.9	14.8	15.7	17.7	19.6	21.5	23.3	25.0	26.6	28.2	29.8
14	-	-						-	-	-	-	14.0	15.0	15.9	16.8	19.0	21.0	22.9	24.8	26.6	28.3	30.0	31.6
16	-	-						-	-	-	-	-	16.0	17.0	17.9	20.2	22.3	24.3	26.2	28.0	29.8	31.6	33.3
18	-	-						-	-	-	-	-	-	18.0	19.0	21.3	23.5	25.6	27.6	29.5	31.3	33.1	34.9
20	-	-						-	-	-	-	-	-	-	20.0	22.4	24.7	26.8	28.9	30.8	32.7	34.6	36.4
25	-	-						-	-	-	-	-	-	-	-	25.0	27.4	29.7	31.9	34.0	36.1	38.0	40.0
30	-	-						-	-	-	-	-	-	-	-	-	30.0	32.4	34.8	37.0	39.2	41.2	43.3
35	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	35.0	37.4	39.8	42.1	44.2	46.4
40	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	40.0	42.5	44.8	47.1	49.3
45	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	45.0	47.5	49.8	52.1
50	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	50.0	52.5	54.8
55	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	55.0	57.5
60	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	60.0



Effective R-Value for Tapered Insulation - Sloped Triangle (Roof with Center Drain)

Rmin ↓ / Rmax →	1	2	3	4	5	6	7	8	9	10	12	14	16	18	20	25	30	35	40	45	50	55	60
1	1.0	1.6	2.2	2.8	3.3	3.9	4.4	5.0	5.5	6.0	7.1	8.2	9.2	10.2	11.3	13.9	16.4	19.0	21.5	24.1	26.6	29.2	31.7
2	-	2.0	2.6	3.3	3.9	4.4	5.0	5.6	6.1	6.7	7.8	8.9	10.0	11.0	12.1	14.7	17.4	20.0	22.6	25.1	27.7	30.3	32.9
3	-	-	3.0	3.7	4.3	4.9	5.5	6.1	6.7	7.2	8.4	9.5	10.6	11.7	12.8	15.5	18.1	20.8	23.4	26.0	28.6	31.2	33.8
4	-	-	-	4.0	4.7	5.3	5.9	6.5	7.1	7.7	8.9	10.0	11.2	12.3	13.4	16.1	18.8	21.5	24.2	26.8	29.5	32.1	34.7
5	-	-	-	-	5.0	5.7	6.3	6.9	7.5	8.1	9.3	10.5	11.7	12.8	13.9	16.7	19.5	22.2	24.9	27.6	30.2	32.9	35.5
6	-	-	-	-	-	6.0	6.7	7.3	7.9	8.6	9.8	11.0	12.2	13.3	14.5	17.3	20.1	22.8	25.6	28.3	30.9	33.6	36.3
7	-	-	-	-	-	-	7.0	7.7	8.3	8.9	10.2	11.4	12.6	13.8	15.0	17.8	20.6	23.4	26.2	28.9	31.6	34.3	37.0
8	-	-	-	-	-	-	-	8.0	8.7	9.3	10.6	11.8	13.0	14.2	15.4	18.3	21.2	24.0	26.8	29.5	32.3	35.0	37.7
9	-	-	-	-	-	-	-	-	9.0	9.7	11.0	12.2	13.4	14.7	15.9	18.8	21.7	24.5	27.3	30.1	32.9	35.6	38.3
10	-	-	-	-	-	-	-	-	-	10.0	11.3	12.6	13.8	15.1	16.3	19.3	22.2	25.1	27.9	30.7	33.5	36.2	39.0
12	-	-						-	-	-	12.0	13.3	14.6	15.9	17.1	20.2	23.1	26.0	28.9	31.8	34.6	37.4	40.2
14	-	-						-	-	-	-	14.0	15.3	16.6	17.9	21.0	24.0	27.0	29.9	32.8	35.6	38.5	41.3
16	-	-						-	-	-	-	-	16.0	17.3	18.6	21.8	24.9	27.9	30.8	33.8	36.7	39.5	42.4
18	-	-						-	-	-	-	-	-	18.0	19.3	22.5	25.7	28.7	31.7	34.7	37.6	40.5	43.4
20	-	-						-	-	-	-	-	-	-	20.0	23.3	26.4	29.5	32.6	35.6	38.5	41.5	44.4
25	-	-						-	-	-	-	-	-	-	-	25.0	28.3	31.5	34.6	37.7	40.7	43.7	46.7
30	-	-						-	-	-	-	-	-	-	-	-	30.0	33.3	36.5	39.7	42.8	45.8	48.9
35	-	-						-	-	-	-	-	-	-	-	-	-	35.0	38.3	41.5	44.7	47.8	50.9
40	-	-						-	-	-	-	-	-	-	-	-	-	-	40.0	43.3	46.5	49.7	52.9
45	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	45.0	48.3	51.6	54.8
50	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	50.0	53.3	56.6
55	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	55.0	58.3
60	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	60.0



Effective R-Value for Tapered Insulation - Sloped Triangle (Perimeter Drains)

Rmin ↓ / Rmax →	1	2	3	4	5	6	7	8	9	10	12	14	16	18	20	25	30	35	40	45	50	55	60
1	1.0	1.3	1.5	1.8	2.0	2.2	2.4	2.5	2.7	2.9	3.2	3.5	3.8	4.1	4.4	5.1	5.8	6.4	7.0	7.6	8.2	8.8	9.3
2	-	2.0	2.3	2.6	2.8	3.1	3.3	3.5	3.7	4.0	4.3	4.7	5.1	5.4	5.8	6.6	7.4	8.1	8.8	9.5	10.2	10.9	11.5
3	-	-	3.0	3.3	3.6	3.9	4.1	4.4	4.6	4.9	5.3	5.7	6.1	6.5	6.9	7.8	8.7	9.5	10.3	11.0	11.8	12.5	13.2
4	-	-	-	4.0	4.3	4.6	4.9	5.2	5.4	5.7	6.2	6.6	7.1	7.5	7.9	8.9	9.8	10.7	11.6	12.4	13.2	14.0	14.7
5	-	-	-	-	5.0	5.3	5.6	5.9	6.2	6.5	7.0	7.5	7.9	8.4	8.8	9.9	10.9	11.8	12.7	13.6	14.4	15.3	16.1
6	-	-	-	-	-	6.0	6.3	6.6	6.9	7.2	7.8	8.3	8.8	9.3	9.7	10.8	11.9	12.8	13.8	14.7	15.6	16.5	17.3
7	-	-	-	-	-	-	7.0	7.3	7.6	7.9	8.5	9.1	9.6	10.1	10.6	11.7	12.8	13.8	14.8	15.8	16.7	17.6	18.5
8	-	-	-	-	-	-	-	8.0	8.3	8.6	9.2	9.8	10.4	10.9	11.4	12.6	13.7	14.8	15.8	16.8	17.8	18.7	19.6
9	-	-	-	-	-	-	-	-	9.0	9.3	10.0	10.5	11.1	11.6	12.2	13.4	14.6	15.7	16.8	17.8	18.8	19.8	20.7
10	-	-	-	-	-	-	-	-	-	10.0	10.6	11.3	11.8	12.4	12.9	14.2	15.4	16.6	17.7	18.7	19.8	20.8	21.7
12	-	-						-	-	-	12.0	12.6	13.3	13.9	14.4	15.8	17.1	18.3	19.4	20.6	21.6	22.7	23.7
14	-	-						-	-	-	-	14.0	14.7	15.3	15.9	17.3	18.6	19.9	21.1	22.3	23.4	24.5	25.6
16	-	-						-	-	-	-	-	16.0	16.7	17.3	18.8	20.2	21.5	22.8	24.0	25.2	26.3	27.4
18	-	-						-	-	-	-	-	-	18.0	18.7	20.2	21.7	23.0	24.3	25.6	26.8	28.0	29.2
20	-	-						-	-	-	-	-	-	-	20.0	21.6	23.1	24.5	25.9	27.2	28.5	29.7	30.9
25	-	-						-	-	-	-	-	-	-	-	25.0	26.6	28.1	29.6	31.0	32.4	33.7	34.9
30	-	-						-	-	-	-	-	-	-	-	-	30.0	31.6	33.2	34.7	36.1	37.5	38.8
35	-	-						-	-	-	-	-	-	-	-	-	-	35.0	36.6	38.2	39.7	41.2	42.6
40	-	-						-	-	-	-	-	-	-	-	-	-	-	40.0	41.6	43.2	44.7	46.2
45	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	45.0	46.6	48.2	49.8
50	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	50.0	51.6	53.2
55	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	55.0	56.6
60	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	60.0